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# Evaluation of a Single-site Daily precipitation Generator in northwestern of Benin

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## Abstract

Stochastic weather generators used in meteorological and hydrological studies are mostly statistical models that produce random numbers resembling the observed data on which they have been fitted. They can generate weather sequences that statistically resemble the real observed data. In this study we developed a single-site daily rainfall generator for the simulations of rainfall occurrences and amounts in northwestern of Benin. A first-order two-state Markov chain was used to determine the occurrence of daily precipitation. The rainfall amounts on wet days were generated by using the one-parameter Exponential distribution and the two-parameter Gamma distribution. The Markov model was successful in simulating the rainfall occurrences and the Exponential distribution as the more reliability in preserving most of the important daily characteristics of the historical rainfall amounts. The test of the two distributions on data that are not taken into account in the elaboration of the model has shown that the exponential distribution reproduces well the statistics of the daily precipitation than the gamma distribution

## Article Info

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## Keywords

Daily rainfall model;
Stochastic process;
Weather generator;
Precipitation;
Benin,
Markov chain

## Introduction

Precipitation is one of the most important meteorological data because of its impact on climate and structures. For example, daily precipitation is required as input of rainfall-runoff models for hydrologists (Obled *et al.*, 2009), used in climatologists' study of droughts (Usman and Reason, 2004; Gitau, 2011; Gitau *et al.*, 2012). In agronomy, they represent the major input variable in plant growth models (Baron *et al.*, 2005; Traore *et al.*, 2011) and empirical statistical models (Schlenker and Lobell, 2010) whose objective is understanding and predicting changes in cereal yields for example (Ramarohetra *et al.*, 2012; Philippon *et al.*, 2015a). Most of these models require long series of daily rainfall. However, the rainfall data observed on many sites are often insufficient or sometimes difficult to be obtained, incompleteness or simply unavailable. This is a serious limitation to the application of agricultural, hydrological and ecosystem simulation models. This has led to the development of mathematical models, known as stochastic rain data generators, frequently used to produce long synthetic meteorological series statistically similar to historical records (Wilks and Wilby, 1999). Rain generators are used as inputs to climate, agricultural, hydrological, and ecological models and are an important tool for studying extreme weather events and risk analysis (Richardson 1985, Semenov and Porter, 1995, Wallis and Griffiths, 1997, Bannayan and Crout, 1999, Brissette *et al.*, 2007, Wilks, 1992, Wilks, 1999).

Stochastic daily rainfall generation approaches on a single meteorological site are available in the hydrological and climatological literature (for example, Richardson, 1981; Srikanthan and McMahon, 1985; Woolhiser, 1992; Sharma and Lall, 1999; Hayhoe, 2000; Wan *et al.*, 2005; Srikanthan *et al.*, 2005; Zheng and Katz, 2008; Liu *et al.*, 2009; Mhanna and Bauwens, 2009). These models are widely used because they are easy to be formulated and quick to be implemented (Wilks, 1999) and are based for the most part on the method of Richardson (1981). It uses a first-order two-state (wet and dry) Markov chain to describe precipitation occurrence and exponential distribution (EXP) (Todorovic & Woolhiser, 1975; Woolhiser & Roldan, 1982), gamma (GAM) (Ison *et al.*, al., 1971, Richardson and Wright, 1984) or a composite distribution, such as the mixed exponential distribution (MEXP) (Woolhiser & Roldan, 1982; Wilks, 1999) to estimate the daily precipitation amount.

The synthetic generation of daily rain on a single site has been conducted in several countries. For example Barkotulla (2010) in Mahadevpur (Bangladesh), Muamaraldin (2009) in the Middle East (Egypt, Jordan and Palestine), and Srikanthan and McMaho (1985) in Australia used the technique suggested by Richardson (1981) and obtained synthetic data statistically similar to that recorded. In West Africa, similar study has not yet been carried out despite the various campaigns it has experienced. The present study aims at establishing a stochastic generation model with daily precipitation data recorded in Northwestern of Benin at Natitingou synoptic station (10.3167°N, 1.3833°E). The study focused on this part of the country because it has a mini-power hydroelectric station installed in Yéripao not far from the station. This mini-power station runs only thanks to the stored rainfall. The knowledge of daily precipitation statistics in this region would therefore improve its design.

## Data and Methods

### Study area and data

Located in North-West Benin (Fig.1) and in the center of the Atacora Department, the town of Natitingou covers an area of 3,045km$^2$. It is located in a valley at the foot of the mountain range of Atacora which rises to 641 meters above sea level. Natitingou is characterized by the Atacora range from where it stands its rugged terrain, composed mainly of plateaus and hills whose valleys are often steep slopes.

Its climate is Sudano-Guinean characterized by two seasons: a wet season that lasts six (06) months (May to October), and a dry season that covers the period from November to April. Due to orographic influences, the Natitingou is very watered sometimes up to 1400 mm per year. Large amounts of rainfall are recorded in August (about 280 mm) and September (Fig.2). The average temperature is about 27°C with variations from 17°C to 35°C during the harmattan period. The river system is reduced to the backwaters, rivers and streams, most of which dry up in the dry season. The most important rivers are: Yeripao, Koumagou and Winmou. The hydrographic network of this city is dependent on climate and relief.

Natitingou weather station (10.3167°N, 1.3833°E) is one of six synoptic stations owned by the National Meteorological Direction of Benin. It's functional since 1921 but the recordings are available from 1950 with some gaps. As part of our study, we considered daily rainfall data regularly recorded from 1970 to 2015. If daily precipitation is higher than 0.1 mm, it is considered as a wet day. Thus, daily precipitation lower than 0.1 mm is regarded as a dry day.

### Methods

The daily precipitation generator can be a Markov chain-Exponential or a Markov chain-Gamma or a mixed Markov chain-Mixed Exponential. A first-order two-state Markov chain is used to determine the occurrence of rainfall.

When a wet day is generated, a two-parameter Gamma distribution, a one-parameter Exponential distribution or a mixed exponential distribution is used to generate the precipitation amount. In this study, the daily precipitation generator is a first-order two-state Markov chain (Larsen and Pense 1982, Roldom, J. and Woolhiser, DA 1982, Richardson 1985) and exponential and gamma distribution are used to generate the precipitation amount.

### Chaîne de Markov

The first-order Markov chain model involves the assumption that the probability of rain on a certain day is conditioned on the wet or dry status of the previous day. Let $X_t$ represent the binary event of precipitation or no precipitation occurring on day t. The process is determined by using the two conditional probabilities for the wet-day occurrence pattern:

$$P_{01} = P[X_t = 1 | X_{t-1} = 0]$$

$$P_{11} = P[X_t = 1 | X_{t-1} = 1]$$

Where the parameters $P_{01}$ and $P_{11}$ are the conditional probability of a wet day ($X_t$=1) following a dry day ($X_t$=0) and the conditional probability of a wet day following a wet day. The complementary probabilities for dry day occurrences are given by $P_{00} = 1 - P_{01}$ and $P_{10} = 1 - P_{11}$ such that, $P_{00} + P_{01} = 1$ and $P_{10} + P_{11} = 1$

The transition probabilities ($P_{01}$ and $P_{11}$) are calculated on all the available recordings in the data set as: $P_{01} = N_{01} / N_0$ and $P_{11} = N_{11} / N_1$ where, $N_{01}$ is the number of wet days after a dry day, $N_{11}$ the number of wet days after a wet day, $N_0$ the total number of dry days and $N_1$ the total number of wet days in the data set.

## Exponential distribution

Exponential distribution is probably the simplest reasonable model for daily precipitation amounts (Wilks and Wilby, 1999). It requires only one parameter β denoted BETA-Exp.

$$f(x) = \frac{1}{\beta} \exp[-x/\beta]$$

The average nonzero precipitation amount according to this distribution is β.

## Gamma distribution

Gamma distribution has been the most popular choice to simulate daily nonzero precipitation amount (Jones *et al.*, 1970, Buishand 1978, Larsen and Pense 1982, Duan et. al. 1995, Wilks 1992, 1999, Danuso 2002, Liao *et al.*, 2004 and Piantadosi et.al. 2008). Its probability density function is

$$f(x) = \frac{(x/\beta)^{\alpha-1} \exp[-x/\beta]}{\beta \Gamma(\alpha)}$$

This distribution involves two parameters: the shape parameter α (denoted ALPHA) and the scale parameter β (BETA). The factor Γ(α) is the gamma function. This distribution has mean M = αβ, and variance $\sigma^2 = \alpha\beta^2$. When α =1, gamma distribution is reduced to exponential distribution.

## Stochastic simulation of daily precipitation

To simulate the precipitation occurrence, a uniform random number between 0 and 1 generated with computer is compared to the appropriate conditional probability $P_{01}$ or $P_{11}$ taking into consideration the wet-dry status of the previous day. If the generated random number is smaller than $P_{01}$, then the previous day is dry followed by a wet day. Alternatively, if the random number is greater than $P_{01}$, then the current day is dry. The decision process is similar if the preceding day is wet. When a wet day is simulated from the occurrence model, precipitation amounts are set. This is done by generating a new random number from a uniform distribution and solving the inverse cumulative distribution of exponential function and the one of gamma function for daily rainfall. Because of seasonal changes of daily precipitation in Northerwest Benin, the conditional propabilities $P_{01}$ and $P_{11}$, the parameter BETA-Exp of exponential distribution, ALPHA and BETA of gamma distribution are calculated respectively in each month with moment method.

## Model Performance Evaluation

The purpose of time generators is to produce synthetic weather data that is statistically similar to that observed. Subsequently, in addition to the graphical comparison, observed and generated precipitation series are subjected to standard exploratory data analysis to describe the performance of the daily generation model using a set of statistical parameters. These statistics are calculated for each month over the 1970-2010 period, ie 41 years:

The mean number of wet days per month;

Mean daily precipitation in a month;

The standard deviation of daily rainfall in a month;

The coefficient of asymmetry of daily precipitation in a month;

Since the models are also used for future events (Stern and Coe, 1982), especially by hydrologists for the design of structures or agronomists for the study of plant growth, daily precipitation over five years were generated and their statistics were compared with those of recorded precipitation over the period 2011-2015.

**Fig.1** Geographical location of North-West Benin with the positioning of the synoptic station of National Direction of Meteorology


**Fig.2** Monthly mean and standard deviation of rainfall recorded at Natitingou from 1970-2015

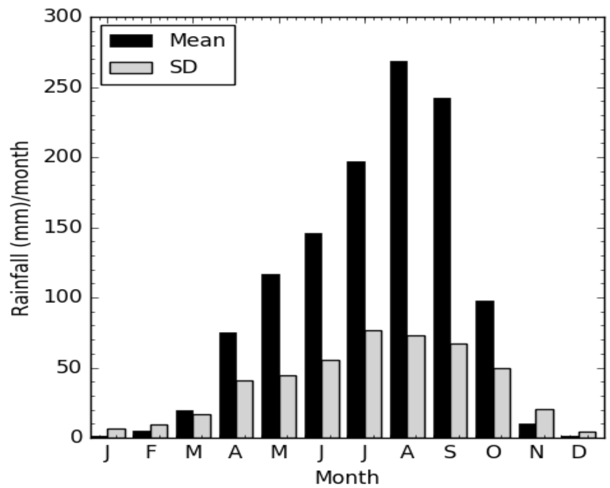


**Fig.3** Conditional probability of daily rainfall from recorded data during 1970 to 2010 at synoptic station of Natitingou

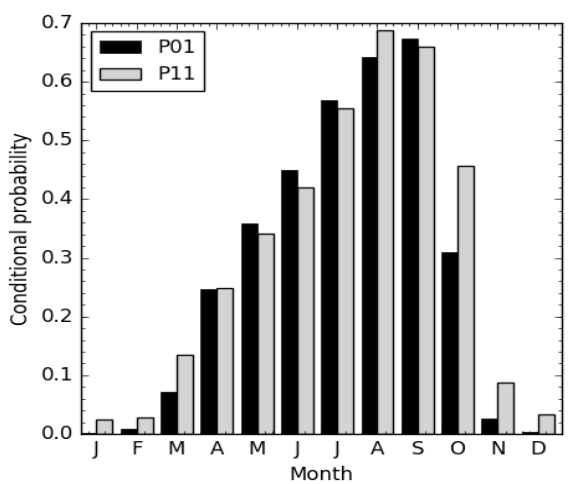

**Fig.4** Statistical comparison of daily precipitation with 41-year simulation and recorded during 1970 to 2010 at synoptic station of Natitingou. (a)-monthly mean wet days; (b)-monthly mean precipitation; (c)-standard deviation of monthly wet days and (d)-standard deviation of monthly rainfall. Gam: gamma distribution and Exp: exponential distribution

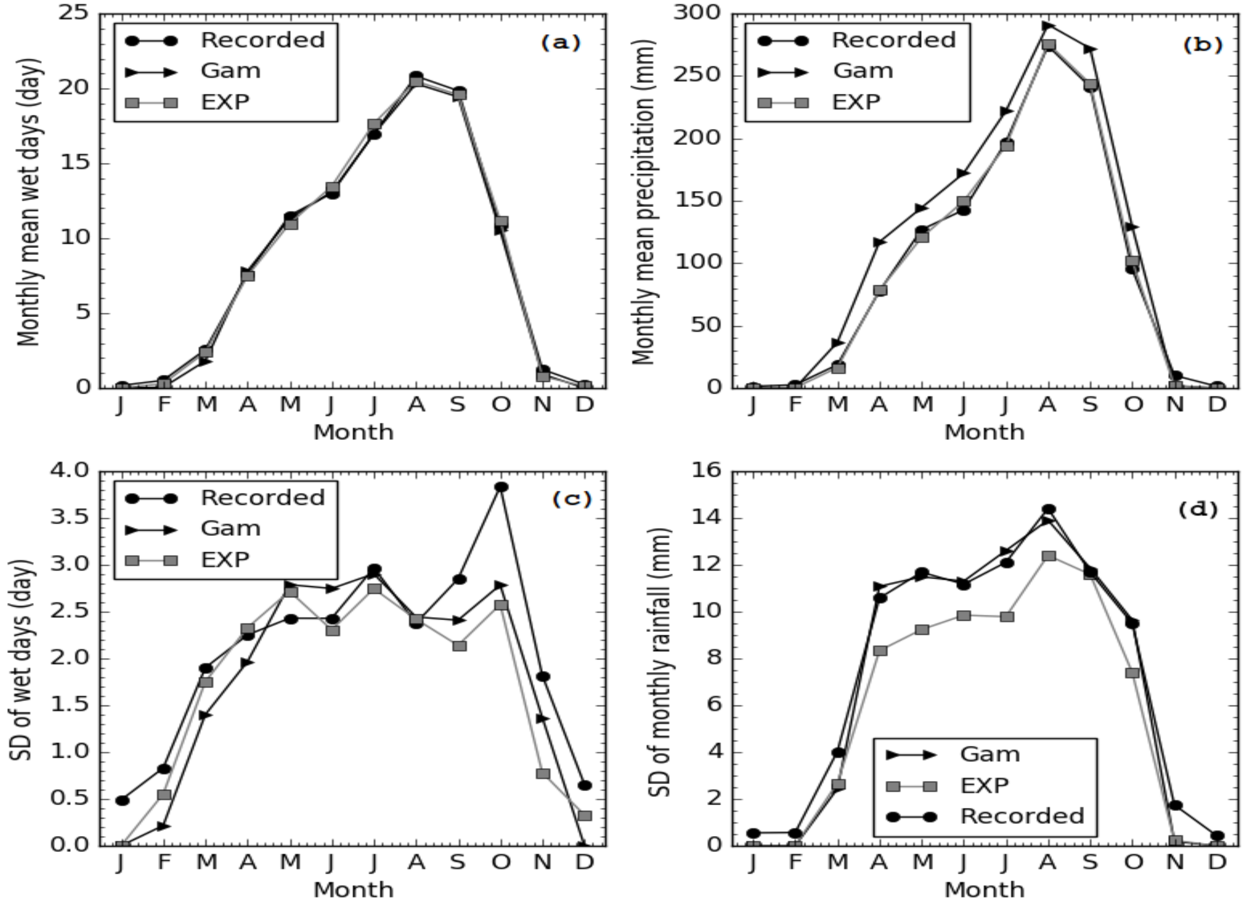


**Fig.5** Statistical comparison of daily precipitation with 41-year simulation and recorded during 1970 to 2010 at synoptic station of Natitingou. (a)-skew coefficient; (b)- mean precipitation in the month. Gam: gamma distribution and Exp: exponential distribution

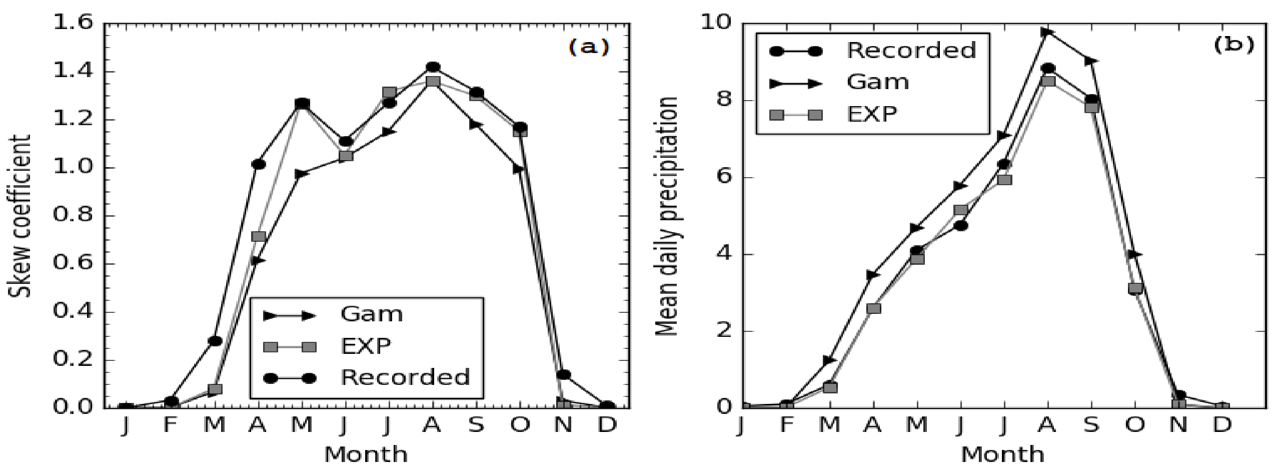

**Fig.6** ECDF comparison of daily precipitation with 5-year simulation and recorded during 1970 to 2010 at synoptic station of Natitingou. (a)-skew coefficient; (b)- mean precipitation in the month. Gam: gamma distribution and Exp: exponential distribution

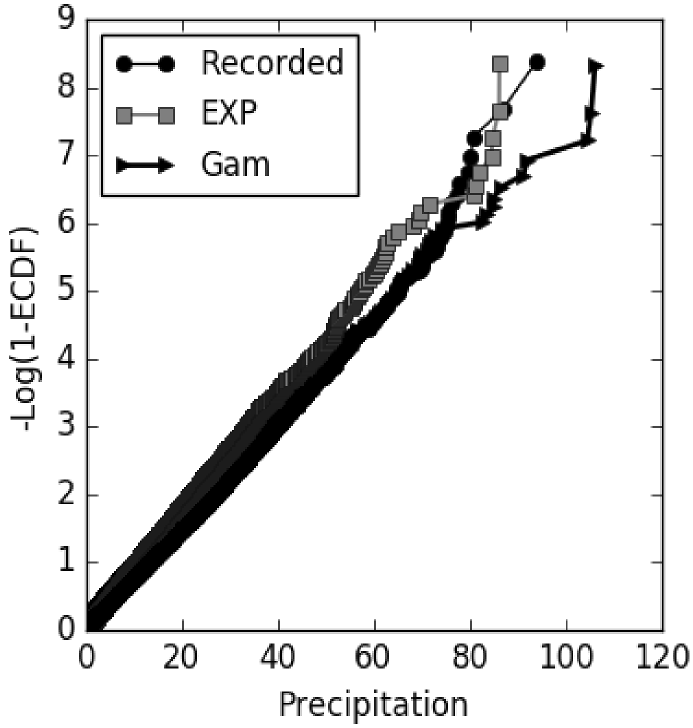


**Fig.7** ECDF comparison of daily precipitation with 5-year simulation and recorded during 2011 to 2015 at synoptic station of Natitingou

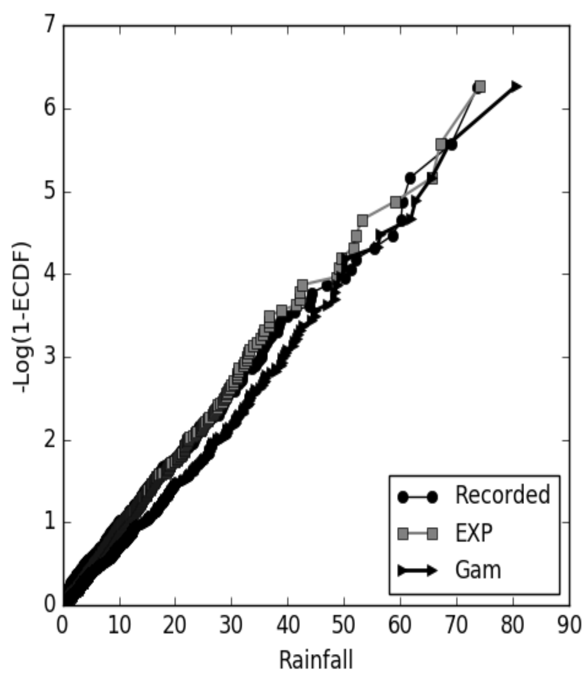

**Table.1** Monthly values of parameters used for the rainfall amounts models

| Month | Exponential | Gamma | |
|---|---|---|---|
| | β | α | β |
| January | 0.57 | 0.05 | 0.54 |
| February | 0.61 | 0.17 | 0.54 |
| March | 5.73 | 4.63 | 4.70 |
| April | 10.18 | 1.19 | 11.08 |
| May | 11.29 | 0.98 | 12.82 |
| June | 11.09 | 1.13 | 12.06 |
| July | 11.45 | 0.97 | 13.28 |
| August | 13.11 | 0.89 | 16.27 |
| Sptember | 12.20 | 1.16 | 11.57 |
| October | 8.66 | 1.14 | 11.10 |
| November | 2.12 | 1.56 | 1.54 |
| December | 0.53 | 0.11 | 0.36 |

## Results and Discussion

### Seasonal evolution of conditional probabilities of the two-state first-order markov chain

Fig.3 shows the monthly change of $P_{01}$ and $P_{11}$ at the synoptic station of Natitingou. The conditional probabilities of wet day rainfall demonstrate the persistence of daily rainfall. During the wet season, a dry day is often more likely to be followed by a wet day except in August and October where a wet day is more likely to be followed by a wet day. During the dry season (November to March) however, the probability of a wet day after a dry day is much lower than a dry day after a dry day. In April particularly, the conditional probabilities of a wet day after a dry or wet day after a wet day are equal. These probabilities show that during the wet period, dry and wet days almost alternate.

### Rainfall Amounts

The parameters employed to generate daily rainfall amounts, using the two distribution methods (exponential and gamma), are presented in Table I. For example, the parameters β required for exponential and gamma distributions increase from January to August, where they peak and decay progressively until December. Parameters of these distributions indicate the significant variation in daily precipitation. Distribution parameters retained for each month were used to simulate daily rainfall. The various daily statistics derived from generated and recorded rainfalls are compared in Fig.4 to Fig. 6.

The results indicate that both models were successful in producing the number of wet days in each month (Fig.4-a). This shows the reliability of the conditional probabilities. The exponential distribution reproduces well the monthly precipitations whereas gamma overestimates them (Fig.4-b). As for the variation in the number of wet days each month, it should be noted that October knows the most variation. Both models try to represent the variation of the other months except that of October. With regard to the variation of the monthly precipitation, the gamma model is better compared to the exponential which underestimates them. In the wet season, the exponential model has almost the same skew coefficients as the recorded precipitation (Fig.5-a). Exponential model reproduces well the daily precipitation while the gamma model overestimate them (Fig.5-b).

In order to highlight the ability of the two models to reproduce slight, moderate and heavy daily precipitation, empirical cumulative distribution function (ECDF) has been shown in Fig.6. This figure shows that the two models simulate well the daily precipitation less than or equal to 50mm. On the other hand daily precipitation between 50 and 80 mm is underestimated by exponential model and well estimated by gamma. Daily precipitation above 80mm are well estimated by exponential whereas gamma model overestimates them. The overestimations noted with the gamma model for both daily and monthly precipitation (Fig.5-b; Fig.4-b) may be justified by this phenomenon highlighted in Fig.6.

Since the models can also be used for future events, five years daily precipitation were generated and compared to daily precipitation recorded from 2011 to 2015 (Fig.7). This figure shows that the exponential distribution produces well recorded rainfall while gamma overestimates them.

This study aimed to establish stochastic generation models of daily precipitation. The two-state, first-order Markov chain was used to determine the occurrence of wet days and exponential and gamma distributions to generate daily precipitation. The results showed that the two-state first-order Markov chain reproduces well the occurrence of wet days. The exponential distribution reproduces well the precipitation whereas gamma overestimates them.

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