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A Common Fixed Point Theorem in Fuzzy Metric Space Using the Control Function

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A B S T R A C T

This paper presents some common fixed point theorem for semicompatiable mapping in fuzzy metric space.

Introduction

The concept of fuzzy sets was introduced by Zadeh [7]. It was developed extensively by many authors and used in various fields to use this concept in topology and analysis several researcher have been defined fuzzy metric space in various ways [3,4,9,10,14,15,16,19]. George and Veeramani [1] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [6] in order to get the Hausdorff topology.Jungck [5] introduced the notion of compatible maps for a pair of self mapping. Cho [17, 18] et.al introduced the concept of semicompatibility of maps. In this paper a fixed point theorem for six self maps has been established using the concept of semicompatibility map which generalizes the result of Cho [17].

Preliminaries

Definition 2.1 [11] A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is called a continuous t norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$ $a,b,c,d\in[0,1]$.

Example of t-norms are $a*b=ab$ and $a*b=\min\{a,b\}$.

Definition 2.2 [1] A Triplet $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,

(i) $M(x, y, 0) = 1$,

(ii) $M(x, y, t) = M(y, x, t)$,

(iii) $M(x, y, t) = 1$ for all t > 0 if and only if $x = y$,

(iv) $M(x, y, \cdot) : [0, \infty) \to [0,1]$ is left continuous,

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(vi) $\lim_{t \to \infty} M(x, y, t) = 1$

Note that $M(x, y, t)$ can be considered as the degree of nearness between x and y with respect to t.We identify $x = y$ with $M(x, y, t) = 1$ for all $t > 0$.

Definition 2.3 [1] Let (X, d) be a metric space. Define $a * b = \min\{a, b\}$ and $M(x, y, t) = \frac{t}{t + d(x, y)}$ for all $x, y \in X$ and all $t > 0$.Then $(X, M, *)$ is a fuzzy metric space.

It is called the fuzzy metric space induced b the metric d.

Definition 2.4 [1] Let (X, M, $*$) be a fuzzy metric space. Then

(i) A sequence $\{x_n\}$ in X is said to converges to x in X if for each $\in > 0$ and each t > 0, there exists $n_0 \in$ N such that $M(x_n, x, t) > 1 - \in$ for all $n \geq n_0$.

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy if for each $\in > 0$ and each t > 0, there exists

$n_0 \in$ N such that M $(x_n, x_m, t) > 1 - \in$ for all $n,m \geq n_0$.

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5[12] Two maps F and G, from a fuzzy metric space (X, M, $*$) into itself are said to be R-weakly commuting if there exists a positive real number R such that for each $x \in X$

$$M(FGx, GFx, Rt) \geq M(Fx, Gx, t) \text{ for all } t > 0 .$$

Definition 2.612] Two maps F and G, from a fuzzy metric space (X, M, $*$) into itself are said to be weak-compatible if they commute at their coincidence points, i.e., $Fx = Gx$ implies $FGx = GFx$

Definition 2.5 [2] A pair (F, G) of self maps of an fuzzy metric space (X, M, $*$) are said to be semi-compatible if $\lim_{n \to \infty} FGx_n = Gx$ whenever $\{x_n\}$ is a sequence such that

$$\lim_{n \to \infty} Fx_n = \lim_{n \to \infty} Gx_n = x \in X .$$

It follows that (F, G) is semi compatible and $Fy = Gy$ then $FGy = GFy$.

Remark 2.8[12] Let (F, G) be a pair of self-maps of a fuzzy metric space (X, M, $*$).Then (F, G) is R-weakly commuting implies that (F,G) is compatible, which implies that (F,G) is weak compatible. But the converse is not true.

Lemma 2.9 [8] Let $(X, M, *)$ be a fuzzy metric space. Then for all $x, y \in X, M(x, y, \cdot)$ is a non-decreasing function.

Lemma 2.10 [18] Let $(X, M, *)$ be a fuzzy metric space. If there exists $k \in (0,1)$ such that for all

$x, y \in X, M(x, y, kt) \geq M(x, y, t)$, $\forall$ $t > 0$, then $x = y$.

Lemma 2.11 [5] Let $\{x_n\}$ be a sequence in a fuzzy metric space $(X, M, *)$. If there exists a number $k \in (0, 1)$ such that $M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t)$ $\forall$ $t > 0$ and $n \in N$ then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.12 [13] The only t-norm $*$ satisfying $r * r \geq r$ for all $r \in [0,1]$ is the minimum t-norm, that is $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$.

Main Result

Theorem 3.1 Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:

(a) $P(X) \subset ST(X)$, $Q(X) \subset AB(X)$,

(b) One of AB or P is continuous,

(c) PB = BP, ST = TS, AB = BA, QT = TQ,

(d) (AB, P) is semicompatible and (Q, ST) is weak compatible,

(e) There exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq \phi\left[\min\left\{M(ABx, STy, t), M(ABx, Px, t), \frac{M(Qy, STy, t) + M(Px, STy, t)}{2}, M(Qy, STy, t)\right\}\right]$$

Where ϕ is continuous function from [0, 1] to [0, 1] such that ϕ(a)> a for each o<a<1 and ϕ(a) =1 and ϕ(0) = 0.

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Let $x_0 \in X$ from (a) there exist $x_1, x_2 \in X$ such that $Px_0 = STx_1 = y_0$ and $Qx_1 = ABx_2 = y_1$.Using by inductively we get a sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Px_{2n} = STx_{2n+1} = y_{2n}$ and $Qx_{2n+1} = ABx_{2n+2} = y_{2n+1}$ for n =0, 1, 2, 3, ...

Step 1 Put $x = x_{2n}$ and $y = x_{2n+1}$ in (e), we get

$$M(\mathrm{P}x_{2n}, Qx_{2n+1}, qt) \geq \phi\left[\min\left\{\begin{array}{l}\mathrm{M}(\mathrm{AB}x_{2n}, \mathrm{ST}x_{2n+1}, \mathrm{t}), \mathrm{M}(\mathrm{AB}x_{2n}, \mathrm{P}x_{2n}, \mathrm{t}), \\ \dfrac{M(Qx_{2n+1}, STx_{2n+1}, \mathrm{t}) + M(Px_{2n}, STx_{2n+1}, \mathrm{t})}{2}, \mathrm{M}(Qx_{2n+1}, STx_{2n+1}, \mathrm{t})\end{array}\right\}\right]$$

$$M\left(y_{2n}, y_{2n+1}, qt\right) \geq \left[\min\left\{M\left(y_{2n-1,}y_{2n}, t\right), M\left(y_{2n-1}, y_{2n}, t\right), \frac{M\left(y_{2n+1,}y_{2n}, t\right) + M\left(y_{2n,}y_{2n}, t\right)}{2}, M\left(y_{2n+1}, y_{2n}, t\right)\right\}\right]$$

$$M\left(y_{2n}, y_{2n+1}, qt\right) \geq \left[\min\left\{M\left(y_{2n-1,}y_{2n}, t\right), M\left(y_{2n-1}, y_{2n}, t\right), \frac{M\left(y_{2n,}y_{2n+1}, t\right) + 1}{2}, M\left(y_{2n}, y_{2n+1}, t\right)\right\}\right]$$

From lemma 2.9 and 2.10 we have

$$M\left(y_{2n}, y_{2n+1}, qt\right) \geq \phi\left[M\left(y_{2n-1,}y_{2n}, t\right)\right]$$

Similarly, we have

$$M\left(y_{2n+1}, y_{2n+2}, qt\right) \geq \phi\left[M\left(y_{2n,}y_{2n+1}, t\right)\right]$$

In general for all n even or odd we have

$$M\left(y_n, y_{n+1}, qt\right) \geq \phi\left[M\left(y_{n-1}, y_n, t\right)\right]$$

$$M\left(y_n, y_{n+1}, t\right) \geq \phi\left[M\left(y_{n-1}, y_n, \frac{t}{q}\right)\right]$$

$$M\left(y_n, y_{n+1}, t\right) \geq \phi\left[M\left(y_{n-1}, y_{n-2}, \frac{t}{q^2}\right)\right]$$

$$\geq -------------$$

$$M\left(y_n, y_{n+1}, t\right) \geq \left[M\left(y_{n-1}, y_{n-2}, \frac{t}{q^n}\right)\right] \to 1, \text{as } n \to \infty$$

Hence M (y_n, y_{n+1}, t) $\to$ 1 as n $\to \infty$ for any t >0 for each ε > 0 and t > 0, we can choose $n_0 \in$ N such that M (y_n, y_m, t) > 1 - ε for all n > n_0.For m, n $\in$ N, we suppose m $\geq$ n.

Then we have

$$\mathrm{M}(y_n, y_m, t) \geq \mathrm{M}\left(y_n, y_{n+1}, \frac{t}{m-n}\right)$$

$$\geq \mathrm{M}\left(y_{n+1}, y_{n+2}, \frac{t}{m-n}\right)$$

$$\geq \mathrm{M}\left(y_{m-1}, y_m, \frac{t}{m-n}\right)$$

$$\geq \frac{(m-n)times}{------------}$$

$$\geq (1 - \varepsilon) * (1 - \varepsilon) * \ldots * (1 - \varepsilon)$$

$$\geq (1 - \varepsilon)$$

Hence $\{y_n\}$ is a Cauchy sequence in X.Since (X, M,$*$) is complete, $\{y_n\}$ converges to some point z $\in$ X. Also its subsequences converge to the same point i.e. z $\in$ X.

i.e. $\{Px_{2n}\} \to z$ and $\{ABx_{2n}\} \to z$ --------- (1)

$\{Qx_{2n+1}\} \to z$ and $\{ST_{2n+1}\} \to z$; ------- (2)

Case I. Let AB is continuous.

$AB(AB)x_{2n} \to ABz$ and $P(AB)x_{2n} \to ABz$

Since (AB, P) is semicompatible therefore we have

$$\lim_{n\to\infty} P(AB)x_{2n} = ABz$$

Step 2

Put $x=ABx_{2n}$ and $y=x_{2n+1}$ in equation (e)

$$M(PAB, Qx_{2n+1}, qt) \geq \phi\left[\min\left\{\begin{array}{l} M(ABABx_{2n}, STx_{2n+1}, t), M(ABABx_{2n}, PABx_{2n}, t), \\ \frac{M(Qx_{2n+1}, STx_{2n+1}, t) + M(PABx_{2n}, STx_{2n+1}, t)}{2}, M(Qx_{2n+1}, STx_{2n+1}, t) \end{array}\right\}\right]$$

Taking $n \to \infty$ we get

$$M(ABz, z, qt) \geq \phi\left[\min\left\{M(ABz, z, t), M(ABz, ABz, t), \frac{M(z,z,t) + M(ABz, z, t)}{2}, M(z,z,t)\right\}\right]$$

$$M(ABz, z, qt) \geq \phi\left[\min\left\{M(ABz, z, t), 1, \frac{1 + M(ABz, z, t)}{2}, 1\right\}\right]$$

$$M(ABz, z, qt) \geq \phi\left[M(ABz, z, t)\right]$$

$$M(ABz, z, qt) \geq \phi\left[M(ABz, z, t)\right] > M(ABz, z, t)$$

There fore using lemma 2.10 we get

$$ABz = z$$

Step 3

Put x=z and y= x_{2n+1} in equation (e)

$$M(Pz, Qx_{2n+1}, qt) \geq \phi\left[\min\left\{\begin{array}{l} M(ABz, STx_{2n+1}, t), M(ABz, Pz, t), \\ \dfrac{M(Qx_{2n+1}, STx_{2n+1}, t) + M(Pz, STx_{2n+1}, t)}{2}, M(Qx_{2n+1}, STx_{2n+1}, t) \end{array}\right\}\right]$$

Taking $n \to \infty$ we get

$$M(Pz, z, qt) \geq \phi\left[\min\left\{M(z, z, t), M(z, Pz, t), \frac{M(z, z, t) + M(Pz, z, t)}{2}, M(z, z, t)\right\}\right]$$

$$M(Pz, z, qt) \geq \phi\left[\min\left\{1, M(Pz, z, t), \frac{1 + M(Pz, z, t)}{2}, 1\right\}\right]$$

$$M(Pz, z, qt) \geq \phi\left[M(Pz, z, t)\right]$$

$$M(Pz, z, qt) \geq \phi\left[M(Pz, z, t)\right] > M(Pz, z, t)$$

There fore using lemma 2.10 we get $Pz = z$.

Therefore $ABz = Pz = z$

Step 4

Put x=Bz and y= x_{2n+1} in equation (e)

$$M(PBz, Qx_{2n+1}, qt) \geq \phi\left[\min\left\{\begin{array}{l} M(ABBz, STx_{2n+1}, t), M(ABBz, PBz, t), \\ \dfrac{M(Qx_{2n+1}, STx_{2n+1}, t) + M(PBz, STx_{2n+1}, t)}{2}, M(Qx_{2n+1}, STx_{2n+1}, t) \end{array}\right\}\right]$$

As BP=PB, AB=BA so we have
$P(Bz) = B(Pz) = Bz$ and $(AB)(Bz) = (BA)(Bz) = B(ABz) = Bz$

Taking $n \to \infty$ and using equation (e)

$$M(Bz, z, qt) \geq \phi\left[\min\left\{M(Bz, z, t), M(Bz, Bz, t), \frac{M(z, z, t) + M(Bz, z, t)}{2}, M(z, z, t)\right\}\right]$$

$$M(Bz, z, qt) \geq \phi\left[\min\left\{M(Bz, z, t), 1, \frac{1 + M(Bz, z, t)}{2}, 1\right\}\right]$$

$$M(Bz, z, qt) \geq \phi\left[M(Bz, z, t)\right] > M(Bz, z, t)$$

Therefore using lemma 2.10 we get $Bz = z$ and therefore also we have $ABz = z$ and this implies that

$Bz = z$. Therefore $Az = Bz = Pz = z - - - -(A-1)$

Step 5

As $P(x) \subset ST(x)$ there exists u $\in$ X such that $z = Pz = STu$

Put x= x_{2n} and y= u in equation (e)

$$M(\mathrm{Px}_{2n}, Qu, qt) \geq \phi\left[\min\left\{\begin{array}{l}\mathrm{M}(\mathrm{ABx}_{2n}, \mathrm{STu}, \mathrm{t}), \mathrm{M}(\mathrm{ABx}_{2n}, \mathrm{Px}_{2n}, \mathrm{t}), \\ \dfrac{M(Qu, STu, \mathrm{t}) + M(P\mathrm{x}_{2n}, STu, \mathrm{t})}{2}, \mathrm{M}(Qu, STu, \mathrm{t})\end{array}\right\}\right]$$

Taking n$\to \infty$ we get

$$M(z, Qu, qt) \geq \phi\left[\min\left\{\mathrm{M}(z, z, \mathrm{t}), \mathrm{M}(z, z, \mathrm{t}), \frac{M(Qu, z, \mathrm{t}) + M(z, z, \mathrm{t})}{2}, \mathrm{M}(Qu, z, \mathrm{t})\right\}\right]$$

$$M(z, Qu, qt) \geq \phi\left[\min\left\{1, 1, \frac{M(Qu, z, \mathrm{t}) + 1}{2}, \mathrm{M}(Qu, z, \mathrm{t})\right\}\right]$$

$$M(z, Qu, qt) \geq \phi\left[\mathrm{M}(Qu, z, \mathrm{t})\right] > \mathrm{M}(Qu, z, \mathrm{t})$$

Therefore using lemma 2.10

We get $Qu = z$

Hence $STu = z = Qu$

Since (Q, ST) is weak compatible there fore $QSTu = STQu$

Thus $Qz = STz$

Step 6

Put x= x_{2n} and y= z in equation (e)

$$M(\mathrm{Px}_{2n}, Qz, qt) \geq \phi\left[\min\left\{\begin{array}{l}\mathrm{M}(\mathrm{ABx}_{2n}, \mathrm{STz}, \mathrm{t}), \mathrm{M}(\mathrm{ABx}_{2n}, \mathrm{Px}_{2n}, \mathrm{t}), \\ \dfrac{M(Qz, STz, \mathrm{t}) + M(P\mathrm{x}_{2n}, STz, \mathrm{t})}{2}, \mathrm{M}(Qz, STz, \mathrm{t})\end{array}\right\}\right]$$

Taking $n \to \infty$

$$M(z, Qz, qt) \geq \phi\left[\min\left\{\mathrm{M}(z, \mathrm{Qz}, \mathrm{t}), \mathrm{M}(z, z, \mathrm{t}), \frac{M(Qz, z, \mathrm{t}) + 1}{2}, \mathrm{M}(Qz, z, \mathrm{t})\right\}\right]$$

$$M(z, Qz, qt) \geq \phi\left[\mathrm{M}(z, \mathrm{Qz}, \mathrm{t})\right] > \mathrm{M}(z, \mathrm{Qz}, \mathrm{t})$$

Therefore using lemma 2.10 we get

$$Qz = z$$

Step7

Put x= x_{2n} and y= Tz in equation (e)

$$M(\mathrm{Px}_{2n}, QTz, qt) \geq \phi\left[\min\left\{\begin{array}{l}\mathrm{M}(\mathrm{ABx}_{2n}, \mathrm{STTz}, \mathrm{t}), \mathrm{M}(\mathrm{ABx}_{2n}, \mathrm{Px}_{2n}, \mathrm{t}), \\ \dfrac{M(Qz, STTz, \mathrm{t}) + M(P\mathrm{x}_{2n}, STTz, \mathrm{t})}{2}, \mathrm{M}(QTz, STTz, \mathrm{t})\end{array}\right\}\right]$$

As $QT = TQ$ and $ST = TS$

We have $QTz = TQz = Tz$ and $ST(Tz) = T(STz) = TQz = Tz$

Taking n$\to \infty$ and we get

$$M(z,Tz,qt) \geq \phi\left[\min\left\{\mathrm{M}(z,\mathrm{Tz},\mathrm{t}),\mathrm{M}(z,z,\mathrm{t}),\frac{M(Tz,Tz,\mathrm{t})+M(z,Tz,\mathrm{t})}{2},\mathrm{M}(Tz,Tz,\mathrm{t})\right\}\right]$$

$$M(z,Tz,qt) \geq \phi\left[\min\left\{\mathrm{M}(z,\mathrm{Tz},\mathrm{t}),1,\frac{1+M(z,Tz,\mathrm{t})}{2},1\right\}\right]$$

Therefore using lemma 2.10

We get $Tz = z$

Now $STz = Tz = z$ implies $Sz = z$

Hence $Sz = Tz = Qz = z - - - - - - - - (A-2)$

Combining equation (A-1 & A-2)

$$Az = Bz = Pz = Qz = Tz = Sz = z$$

Hence z is the common fixed point of A, B, S, T, P, and Q.

Case-II Let P is continuous

$PPx_{2n} \to Pz$ and $P(AB)x_{2n} \to Pz$.

Since (AB,P) is semicompatible therefore we have

$$\lim_{n\to\infty} AB(\mathrm{P})\,\mathrm{x}_{2n} = Pz$$

Step 8

Putting $x = Px_{2n}$ and $y = x_{2n+1}$ in equation (e) we have

$$M(PPx_{2n},Qx_{2n+1},qt) \geq \phi\left[\min\left\{\begin{matrix}M(ABPx_{2n},STx_{2n+1},t),M(ABPx_{2n},\mathrm{PPx}_{2n},t),\\ \frac{M(Qx_{2n+1},STx_{2n+1},t)+M(PPx_{2n},STx_{2n+1},t)}{2},M(Qx_{2n+1},STx_{2n+1},t)\end{matrix}\right\}\right]$$

Taking $n \to \infty$

$$M(\mathrm{Pz},z,qt) \geq \phi\left[\min\left\{M(Pz,z,t),M(Pz,\mathrm{Pz},t),\frac{M(z,z,t)+M(Pz,z,t)}{2},M(z,z,t)\right\}\right]$$

$$M(\mathrm{Pz},z,qt) \geq \phi\left[\min\left\{M(Pz,z,t),1,\frac{1+M(Pz,z,t)}{2},1\right\}\right]$$

$$M(\mathrm{Pz},z,qt) \geq \phi\left[\mathrm{M}(Pz,z,t)\right] > M(Pz,z,t)$$

Therefore using lemma 2.10 we get

$$Pz = z$$

Using step 5, 6 and 7 we get

$$Qz = STz = Sz = TZ = z$$

Step 9

As $Q(X) \subset AB(X)$ there exists $w \in X$ such that

$$z = Qz = ABw$$

Put $x = w$ and $y = x_{2n+1}$ in equation (e) we have

$$M(Pw,Qx_{2n+1},qt)\geq\phi\left[\min\left\{\begin{array}{l}M(ABw,STx_{2n+1},t),M(ABw,Pw,t),\\ \dfrac{M(Qx_{2n+1},STx_{2n+1},t)+M(Pw,STx_{2n+1},t)}{2},M(Qx_{2n+1},STx_{2n+1},t)\end{array}\right\}\right]$$

Taking $n\to\infty$

$$M(Pw,z,qt)\geq\phi\left[\min\left\{M(z,z,t),M(z,Pw,t),\frac{M(z,z,t)+M(Pw,z,t)}{2},M(z,z,t)\right\}\right]$$

$$M(Pw,z,qt)\geq\phi\left[\min\left\{M(z,z,t),M(Pw,z,t),\frac{M(z,z,t)+M(Pw,z,t)}{2},M(z,z,t)\right\}\right]$$

$$M(Pw,z,qt)\geq\phi\left[\min\left\{1,M(Pw,z,t),\frac{1+M(Pw,z,t)}{2},1\right\}\right]$$

$$M(Pw,z,qt)\geq\phi\left[\mathrm{M(Pw,z,t)}\right]>M\left(Pw,z,t\right)$$

Therefore by using lemma 2.10 we get

$$Pw=z$$

Therefore

$$ABw=Pw=z\,.$$

As (AB, P) is semicompatible map.
We have

$$Pz=ABz$$

Also from step 4 we get

$$Bz=z$$

Therefore $Az=Bz=Pz=z$ and we see that z is common fixed point of the six maps in this case also.

Step 10

Uniqueness: let u be another common fixed point of A, B, S, T, P and Q
Then $Au=Bu=Pu=Qu=Su=Tu=u$
Put $x=z$ and $y=u$ in equation (e) we get

$$M(\mathrm{Pz},Qu,qt)\geq\phi\left[\min\left\{\mathrm{M(ABz,STu,t),M(ABz,Pz,t)},\frac{M(Qu,STu,\mathrm{t})+M(Pz,STu,\mathrm{t})}{2},\mathrm{M}(Qu,STu,\mathrm{t})\right\}\right]$$

Taking $n\to\infty$ we get

$$M(\mathrm{z},u,qt)\geq\phi\left[\min\left\{\mathrm{M(z,u,t),M(z,z,t)},\frac{M(u,u,\mathrm{t})+M(z,u,\mathrm{t})}{2},\mathrm{M}(u,u,\mathrm{t})\right\}\right]$$

$$M(\mathrm{z},u,qt)\geq\phi\left[\mathrm{M(z,u,t)}\right]>\mathrm{M(z,u,t)}$$

Therefore by using lemma 2.10 we get $z=u$ .There fore z is the unique common fixed point of semicompatable A, B, S, T, P, and Q.

Corollary 3.2 Let $(X,M,*)$ be a complete fuzzy metric space and A, S, P and Q be mapping from X into itself such that the following condition are satisfied.

(i) $P(X)\subset S(X),Q(X)\subset A(X)$,

(ii) One of A or P is continuous,

(iii)(A,P) is semicompatible and (Q,S) is weak compatible,

(iv)There exists $q \in (0,1)$ for every $x, y \in X$ and $t > 0$,

(v) $M(Px, Qy, qt) \geq \phi\left[\min\left\{M(Ax, Sy, t), M(Ax, Px, t), \frac{M(Qy, Sy, t) + M(\mathrm{Px}, \mathrm{Sy}, \mathrm{t})}{2}, M(Qy, Sy, t)\right\}\right]$

Then A, S, P and Q have a unique common fixed point in X.

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